Midterm Exam

Last name	First name	SID

Rules.

- You have 80 mins (3:40pm - 5pm) to complete this exam.
- The exam is not open book, but you are allowed half a sheet of handwritten notes; calculators will be allowed.
- No form of collaboration between the students is allowed. If you are caught cheating, you may fail the course and face disciplinary consequences.

Please read the following remarks carefully.

- Show all work to get any partial credit.
- Take into account the points that may be earned for each problem when splitting your time between the problems.

Problem	Points earned	out of
Problem 1		15
Problem 2		50
Problem 3		15
Problem 4		20
Total		100

Problem 1 [15] Answer the following questions briefly but clearly. **You must justify your T/F answer.**

(a) [3] True/False: If X and Y are random variables then $\mathrm{var(X+Y)} \geq \max\{\mathrm{var(X)}, \mathrm{var(Y)}\}$

(b) [3] True/False: Let X be a normal random variable with mean a^2 and variance b^4, and let $Y = a^2X + b^3$. Then the random variable $Z = \frac{1}{10}\frac{Y-a^4-b^3}{a^2b^2}$ is a normal random random variable with mean zero and variance $.01$.

(c) [3] True/False: If X and Y are independent exponential random variables with parameters λ_x and λ_y respectively, then $\min\{X, Y\}$ is an exponential random variable with parameter $\min\{\lambda_x, \lambda_y\}$

(d) [3] True/False: If two events are dependent they cannot be conditionally independent.

(e) [3] True/False: For any continuous random variable that has finite expectation, $E[X]$: $P(X \leq E[X]) = 0.5$.

Problem 2[50] Answer the following questions briefly but clearly.

(a) [5] A couple has two children. The probability that they will conceive a boy =0.5 and births are independent of each other. If we are told that their second born is a boy, what is the probability that the first born is a boy?

(b) [5] Three persons each rolls a fair 6-sided die once. Let A_{ij} be the event that person i and person j roll the same face. Are the events A_{12}, A_{13}, and A_{23} jointly independent? Are they pairwise independent?

(c) [10] The letters E,E,P,P,P,R are stamped on tiles and put in a bag. They are drawn from the bag without replacement one by one at random. What is the probability that the letters are drawn in the sequence P,E,P,P,E,R? How many different permutations of letters are there that begin with E?

(d) [10] Alice chooses a random number x uniformly between 0 and 1. Bob then picks independent random numbers $y_1, y_2...,$ uniformly from $[0, 1]$ until he has picked a number $y_L > x$. (So $y_i < x$ for $i = 1, 2, ..., L - 1$.) Bob is then given $L - 1$ dollars. What are Bob's expected winnings, I.e., what is $E[L - 1]$? HINT: What is $P(L > n)$?

(e) [10] Suppose there are n pairs of socks in the dryer and you pull out k socks at random. What is the average number of pairs will you have?

(f) [10] If independent trials each resulting in success with probability p are performed, what is the probability of r successes before m failures?

Problem 3 [15] Two jars contain 4 balls each. Jar 1 has red balls and Jar 2 has blue balls. A "switch" consists of randomly selecting a ball from each jar and simultaneously moving it into the other jar.

(a) [10] For a given ball, b, let S_b be the total number of times it is selected in 4 switches. Find the pmf and expected value of S_b.

(b) [5] What is the expected number of balls that are in their original jars after 4 switches?

Problem 4 [20] Alice visits an island where there are two kinds of people: Alphas and Betas. Alpha's tell the truth with probability p_1 and Betas tell the truth with probability p_2. The probability that a randomly chosen person is an Alpha is α. Note that the responses from islanders are independent of each other.

(a) [5] Alice cannot tell the two types apart and so decides to test a randomly chosen islander with questions she knows the answer to. She asks this person n questions and receives k correct responses. What is the probability that she has chosen an Alpha?

(b) [10] Alice chooses another person at random and asks him a "Yes/No" question. The person answers "Yes". She then asks another randomly chosen person if the first person was telling the truth and this person says "Yes". What is the probability that the first person told the truth?

(c) [5] Alice finally figures out how to tell the Alphas and Betas apart. She asks a fixed "Yes/No" question of N different Alphas. Given that they all give her the same answer, what is the probability that they are being truthful?