UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Problem Set 7
Spring 2011

Issued: Tuesday, March 8, 2011 **Due:** In HW box Wednesday, Mar 16, 2011

Reading: Bertsekas & Tsitsiklis, §4.1–4.5

Problem 1. Assume a lifetime of a bulb is an exponential random variable with a parameter λ. The bulb factory has two production lines. The first line is twice as fast as the second one, but the bulbs from the first line have shorter lifetime ($\lambda_1 > \lambda_2$).

(a) What is the probability that a randomly chosen bulb comes from the first production line?

(b) If it is still on after t hours, what is the probability that the bulb came from the first production line?

(c) After how many hours will it be more likely to come from the second production line?

(d) An office building gets all of its bulbs from the first production line. Because of the maintenance schedules, the cost associated with changing a bulb is proportional to the deviation of the bulb's lifetime from a hours (if it burns out too early, a special delivery must be ordered; if it stays on for longer, it is a waste of time and resources to change it after a hours). Find the PDF of the maintenance cost associated with each bulb. What is the expected cost?

Problem 2. The transform and the mean of a random variable X are given by $M_X(s) = ae^s + be^{13(e^s-1)}$ and $E[X] = 5$ respectively. Using properties of the moment generating function find the numerical values of:

(a) The constants a and b.

(b) $E[e^{5X}]$.

(c) $P(X = 1)$.

(d) $E[X^2]$.

Problem 3. Suppose X is a geometric random variable with parameter $1 - P$ where P is uniformly distributed from 0 to $\frac{n-1}{n}$. Define a new random variable Z by $Z = E[X|P]$. Find and interpret $E[Z]$, and then compute $\lim_{n\to\infty} E[Z]$.

Problem 4. Suppose $X \sim N[0,1]$ and $Z = 0$ or 1 with equal probability. Now consider a random variable Y such that:

$$Y = \begin{cases} X & \text{if } Z = 1 \\ -X & \text{if } Z = 0 \end{cases}$$

(a) Are X, Y independent?

(b) Are Y, Z independent?

(c) Show that $Y \sim N[0,1]$.

(d) Show that $Cov(X,Y) = 0$.

Problem 5. **The Kelly strategy** Consider a gambler who at each gamble either wins or loses his bet with probabilities p and $1-p$, independently of earlier gambles. When $p > 1/2$, a popular gambling system, known as the Kelly strategy, is to always bet the fraction $2p-1$ of the current fortune. Assuming $p > 1/2$, compute the expected fortune after n gambles of a gambler who starts with x units and employs the Kelly strategy. HINT: Consider X_n, the *fraction* at time n of the original fortune x. By linearity of expectation the expected fortune is $x \cdot X_n$..

Problem 6. Consider n independent tosses of a die. Each toss has probability p_i of resulting in i. Let X_i be the number of tosses that result in i. Show that X_1 and X_2 are negatively correlated (i.e., a large number of ones suggests a smaller number of twos).

Problem 7. Using a fair three-sided die (construct one, if you dare), we will decide how many times to spin a fair wheel of fortune. The wheel of fortune is calibrated infinitely finely and has numbers between 0 and 1. The die has the numbers 1,2 and 3 on its faces. Whichever number results from our throw of the die, we will spin the wheel of fortune that many times and add the results to obtain random variable Y.

(a) Determine the expected value of Y.

(b) Determine the variance of Y.